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ÚGN

INVITATION TO THE LECTURE

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EFFICIENT KARHUNEN-LOÈVE EXPANSIONS FOR GAUSSIAN RANDOM FIELDS VIA LEGENDRE-GALERKIN DISCRETIZATION AND TENSOR STRUCTURE

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This talk presents an efficient framework for computing Karhunen-Loève (KL) expansions of stationary isotropic Gaussian random fields on hyper-rectangular domains. Since standard Galerkin discretizations of the KL eigenproblem lead to dense operators, the main goal is to reduce both assembly cost and the cost of the resulting eigenvalue computations. The approach begins with a positive approximation of the covariance kernel by a finite sum of squared exponentials, which makes the kernel separable. This in turn induces a tensor-product structure in a Legendre-Galerkin discretization, allowing the discrete operator to be represented by Kronecker products of one-dimensional blocks. Additional savings are obtained through an even/odd parity block decomposition and a stable Duffy-type quadrature for the required integrals. The talk outlines the main theoretical motivation for the positive Gaussian-mixture approximation, describes the computational pipeline, and illustrates the method on numerical results for covariance approximation, eigenvalue decay, random field sampling, and error behavior. Overall, the presented framework shows how approximation theory, tensor structure, and stable numerical integration can make KL expansions substantially more efficient while maintaining accuracy.