Institute of Geonics, Czech Academy of Sciences
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INVITATION TO THE LECTURE

MARCH 4, 2026 12:30 ROOM 213

NONLINEAR ENERGY MINIMIZATION WITH FEM AND AUTOMATIC DIFFERENTIATION

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This talk presents a practical workflow for solving nonlinear variational problems with FEM by specifying only the energy functional $J(u)$. Automatic differentiation provides the exact gradient $\nabla J(u)$ and the full Hessian $\nabla^2 J(u)$ without hand-derived formulas and without fragile finite-difference tuning.

A central topic is how to reconstruct an explicit sparse Hessian from a `HessMult` interface. We combine (i) the FEM-induced sparsity pattern (P1 elements), (ii) graph coloring on H^2, and (iii) a small number of Hp evaluations to recover all nonzeros. This yields analytic second-order information at a cost comparable to sparse finite differences, with improved robustness.

We discuss which problem classes fit this framework (p-Laplacian, Ginzburg--Landau/phase-field models, Neo-Hookean hyperelasticity and related local energy densities) and illustrate it on the p-Laplacian, Ginzburg--Landau/phase-field models, and Neo-Hookean hyperelasticity. We then focus on a practical recipe: Newton's method with line search or trust-region (and possible hybrids), an appropriate iterative method for the linear subproblems, and a matching preconditioner.